CSE 307: Principles of Programming Languages

Logic Programming

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Section 1

Logic Programming

Topics

1. Logic Programming

Logic and Programs

- "All men are mortal; Socrates is a man; Hence Socrates is mortal"

$$\forall X.\ man(X) \Rightarrow mortal(X)$$

$$man(\texttt{socrates})$$

- Predicate logic
  - Predicates (e.g. man, mortal) which define sets.
  - Atoms (e.g. `socrates`) which are data values
  - Variables (e.g. X) which range over data values
  - Rules (e.g. $\forall X.\ man(X) \Rightarrow mortal(X)$) which define relationships between predicates.
-

```
mortal(X) :- man(X).
man(socrates).
```

```
let isMortal(x) = isMan(x);;
let isMan(x) = (x = socrates);;
```

Logic Programs

```
mortal(X) :- man(X).
man(socrates).
```

```
?- mortal(socrates).
yes
?- mortal(X).
X=socrates ;

no
```

Relations and Logic Programs

- Unary predicates (e.g. `man`, `mortal`) define *sets*.

  Predicates with higher arity (binary, ternary etc) define *relations*. Example:

  ```
  flight(jfk, dfw).        flight(stl, jfk).
  flight(dfw, lax).        flight(stl, dfw).
  flight(lga, stl).
  ```

- **Facts:** sets and relations whose definitions do not depend on anything else. (e.g. `man(socrates)`).

  “extensional data base” (EDB)

Relations and Logic Programs (Contd.)

- **Rules** define *computed* sets and relations (e.g. `mortal`).

  "intensional data base" (IDB) relations

  ```
  canFly(Source, Dest) :- flight(Source, Dest).
  canFly(Source, Dest) :- flight(Source, Stopover),
          canFly(Stopover, Dest).
  ```

Programming with Logic

- Data structures:
  - Atomic data such as `socrates`, `lga`, etc.
  - Data structures by constructing *terms* (tree structures):
    - `[]`: nil list
    - `[X|Xs]`: list with `X` as its head and `Xs` as its tail
    - `prog(P, D, S)`: a structure with `prog` as the *root* symbol, and `P`, `D`, and `S` as its children
- Example programs: `append(Xs,Ys,Zs)`: `Xs`, `Ys`, and `Zs` are lists such that `Zs` is the contactenation of `Xs` and `Ys`.

```
 append([], Ys, Ys).
append([X|Xs], Ys, [X|Zs]) :-
    append(Xs, Ys, Zs).
```

From Functional to Relational Programming

```
let rec append(l, ys) =
  match l with
    [] -> ys
   x::xs -> x::append(xs, ys)
```

```
append([], Ys, Z) :- Z=Ys.
append([X|Xs], Ys, Z) :-
   append(Xs, Ys, Zs),
   Z = [X|Zs].
```

```
append([], Ys, Ys).
append([X|Xs], Ys, [X|Zs]) :-
   append(Xs, Ys, Zs).
```

```
let rec reverse l =
  match l with
    [] -> []
   x::xs ->
   append((reverse xs), [x])
```

```
reverse([], Z) :- Z=[].
reverse([X|Xs], Z) :-
   reverse(Xs, T),
   append(T, [X], Z).
```

SML and Prolog

```
fun rev1(x::xs, ys) =
     rev1(xs, x::ys)
|   rev1(nil, ys) = ys
fun rev(xs) = rev1(xs, [])


 datatype tree =
   Node of int * tree * tree
 | Leaf of int;
 fun search(Node(i,l,r), j) =
       if (j<=i) then search(l,j)
       else search(r,j)
 |   search(Leaf(i), j) = i = j;
```

```
rev1([X|Xs], Ys, Zs) :-
  rev1(Xs, [X|Ys], Zs)
rev1([], Ys, Ys).
rev(Xs, Ys) :- rev1(Xs,[],Ys)




search(node(I,L,R), J) :-
  (J =< I -> search(L, J);
     search(R, J)).
search(leaf(I),I).
```

Syntax of Prolog Programs

- *Names:*
  - Variable names start with uppercase letters
  - Predicate names start with lowercase letters
  - Data constructors (called “function symbols” and “constants”) start with lowercase letters *or enclosed in single quotes*
- *Data structures:* a *term* (a tree of symbols) built using function symbols *and variables*.
  - `lga`
  - `[1]` (same as `[ 1 | [ ] ]`)
  - `[1,2]` (same as `[1 | [ 2 | [ ] ] ]`)
  - `f(g(a))`
  - `f(g(h(X)))`
  - `f(X, g(X))`
  - `(lga, jfk)`

Syntax of Prolog Programs (Contd.)

- *Atom:* a term built with function symbols, predicate symbols and variables.

  Example: `append([X|Xs], Ys, [X|Zs])`

- *Clauses:* of the form $lhs :- rhs$.

  Note the trailing period.

  - Clause head: An atom
  - Clause body: a comma-separated sequence of atoms.
  - Facts: clauses with empty bodies.

    Written as lhs.
  - Rules: clauses with non-empty bodies.

- *Program:* a sequence of clauses.

- *Query:* an atom.

Arithmetic in Prolog

- Use of "=" simply constructs or inspects term structures.
  - For example, `X = 1 + 2` binds `X` to term `1+2`.
- Binary operator "`is`" should be used to *evaluate* arithmetic expressions.
  - For example, `X is 1 + 2` binds `X` to `3`.
  - Rhs of "`is`" must be *ground* when the operator is evaluated.
- Expressions mix real and integer arithmetic, lifting values to real whenever necessary.
- Arithmetic comparison operators: `=`, `¯`, `<`, `>`, `=<`, `>=` (Note the syntax of "less-than-or-equal-to" etc.)
-
  ```
  length([], 0).
  length([X|Xs], N) :- length(Xs, M), N is M+1.
  ```

How Prolog Works

Prolog attempts to check if the given query q is true by

1. Is there a clause whose left hand side corresponds to q?
2. If not, q is false (we say that q fails)
3. If there is such a clause, say $l :- r_1, r_2, \ldots, r_n$
    - Now check if *all of* $r_1, r_2, \ldots$ are true.
    - If so, q is true (we say that q succeeds)
    - If not, repeat step (3) until there is no matching clause

- Clauses are tried in the order they appear in the program.
- If more than one clause applies, *they are tried one after another* until the goal succeeds

How Prolog Works (Contd.)

```
append([], Ys, Ys).
append([X|Xs], Ys, [X|Zs]) :-
   append(Xs, Ys, Zs).
```

`append([a,b], [c], Z)`	Clause 2
`append([b], [c], Z’), Z = [a|Z’]`	Clause 2
`append([], [c], Z’’), Z’=[b|Z’’], Z = [a|Z’]`	Clause 1
`Z’’=[c], Z’=[b|Z’’], Z = [a|Z’]`	Simplify
`Z=[a,b,c]`	

How Prolog Works (Contd.)

```
append([], Ys, Ys).
append([X|Xs], Ys, [X|Zs]) :-
   append(Xs, Ys, Zs).
```

`append(U, V, [a,b])`	Clause 1, Clause 2
`(1) U=[], V=[a,b]`	
`(2) append(U’,V,[b]), U=[a|U’]`	Clause 1, Clause 2
`(2.1) U’=[], V=[b], U=[a|U’]`	Simplify
`U=[a], V=[b]`	
`(2.2) append(U”,V,[]), U’=[b|U”], U=[a|U’]`	Clause 1
`U”=[], V=[], U’=[b|U”], U=[a|U’]`	Simplify
`U=[a,b], V=[]`	

Unification

- *Unification* is the operation to make two data structures identical (i.e. “unify” them). Predefined binary predicate `=` may be used to unify terms.
  - `a = a` succeeds, `a = b` fails, `X = a` succeeds after binding `X` to `a`.
  - `f(X) = f(a)` succeeds after binding `X` to `a`.
  - `g(a) = f(a)`, `f(a) = f(b)`, `f(a,b) = f(b,a)` fail.
  - `?- f(X) = f(a), X = b.`
  - `?- f(X,a) = f(b,Y).`
  - `?- f(X,a) = f(b,X).`
- A clause is applicable if the query (also called a *goal* or *subgoal*) **unifies** with the left hand side of the clause.

Unification (Contd.)

- *Substitution:* a function that maps variables to *values* (terms).
- An *unifier* of two terms t_1 and t_2 is a substitution over variables of t_1 and t_2 that make them identical.
  - The substitution $\{\mathtt{X} \rightarrow \mathtt{b}, \mathtt{Y} \rightarrow \mathtt{a}\}$ is an unifier of `f(X,a)` and `f(b,Y)`.
  - The substitution $\{\mathtt{X} \rightarrow \mathtt{b}, \mathtt{Y} \rightarrow \mathtt{a}, \mathtt{Z} \rightarrow \mathtt{c}, \mathtt{W} \rightarrow \mathtt{c}\}$ is an unifier of `f(X,a,Z)` and `f(b,Y,W)`.
  - The substitution $\{\mathtt{X} \rightarrow \mathtt{b}, \mathtt{Y} \rightarrow \mathtt{a}, \mathtt{Z} \rightarrow \mathtt{d}, \mathtt{W} \rightarrow \mathtt{d}\}$ is an unifier of `f(X,a,Z)` and `f(b,Y,W)`.
  - The substitution $\{\mathtt{X} \rightarrow \mathtt{b}, \mathtt{Y} \rightarrow \mathtt{a}, \mathtt{Z} \rightarrow \mathtt{W}\}$ is an unifier of `f(X,a,Z)` and `f(b,Y,W)`.

    Called the *most general unifier*

    During query evaluation, clauses are selected by computing the most general unifier.

A Simple Prolog Interpreter: Types

```
type nonvar = string
type var = int
type term = Var of var | Nvar of nonvar * term list
type clause = term list
type goal = term
type program = clause list

type subst = (var * term) list
type env = int (* base pointer *) * subst

type path = goal list * env
```

A Simple Prolog Interpreter: unify

```
let rec unify: subst -> term -> term -> subst =
  fun subst t1 t2 = match (t1, t2) with
  | (Var(x), _) -> add_subst subst x t2
  | (_, Var(y)) -> add subst y t1
  | (Nvar(c,t1s), Nvar(d,t2s)) ->
       if c=d then unify_list subst t1s t2s
       else raise Unif_fail

and unify_list subst l1 l2 = fold_left2 unify subst l1 l2

and add_subst: subst->var->term->subst = fun subst x t =
  try let t' = assoc x subst in unify subst' t' t
  with Not_found -> if t<>Var(x) then (x,t)::subst else subst
```

More about unification ...

- Given two terms t_1 and t_2 containing variables $\overline{x}_1$ and $\overline{x}_2$, t_1 and t_2 are unifiable if and only if the logical formula $\boxed{\exists \overline{x}_1 \overline{x}_2\ t_1 = t_2}$ is satisfiable.
- Unification procedure computes a solution to the formula, i.e., a valuation for $\overline{x}_1$ and $\overline{x}_2$ that makes this formula true.
- Every solution to the formula is an instance of the solution computed by `unify` — the *most general unifier* property.
- *Occurs-check:* Note that $\forall X \;\; X \neq f(X)$.
  - So, in general, we need to check if X *occurs* in t before taking t as a substitution for X.
  - Omitted in Prolog because it has severe impact on performance
  - Interestingly, `unify` terminates even when it computes such cyclic substitutions!

More about unification ... (Continued)

- *Unification* is a *constraint-solving procedure* for equality constraints over terms.
- Many problems can be modeled in terms of such constraints

  Type inference:

  - For each identifier i, associate a variable T_i that holds its type.
  - Constraints on T_i's types are inferred from each use of i, whether it be as argument to a function, in an equality or match operation, etc.
  - Most general unifiers yield the most general types for each identifier.

  Logic program evaluation:

  - Each “call” introduces a constraint between actual and formal parameters.
  - Most general unifiers correspond to the most general solutions to the query

Type Inference Example

```
let h y = 0

let g x =
  if (l x)
    then (h x)
  else (g (x+1))

let rec f t =
  match t with
  | [] -> []
  | z::zs -> (g z)::(f zs)
```

$T_h : T_y \to int$

$T_x : in(T_l)$

$T_g : T_x \to out(T_h, T_x)$

$T_g : int \to out(T_g, int),\ T_x : int$

$T_t : \alpha\ list$

$T_f : T_t \to \beta\ list$

$T_f : T_t \to out(T_g, \alpha) list$

$T_f : T_t \to out(T_f, T_t)$

Query evaluation in Prolog

- The query evaluation procedure in Prolog (called clause resolution) uses *backtracking* search.
- Given a query (goal), a clause is *applicable* if its head (lhs) unifies with the query.
- When more than one clause is applicable evaluation,
  - the first clause is selected, and query evaluation continues with the body of the clause
  - ... but we may come back to try the remaining clauses if further query evaluation using the first clause fails.
- Clauses applicable but not yet tried at any point are remembered *and are tried upon backtracking.*
- *Alternative strategy:* Eagerly compute all solutions
  - Let us write a simple interpreter for this strategy

A simple Prolog interpreter to compute all solutions

```
let rec call: (prog: clause list) (env:env) (goal:goal): env list =
  let paths = (map (find_path goal env) prog) in
  let viable_paths = filter (fun (_, (bp, _)) -> bp > 0) paths
  in exec_paths prog viable_paths

and exec_paths prog paths = match paths with
  | [] -> []
  | p1::ps -> (append (exec_path prog p1) (exec_paths prog ps))

and exec_path: program -> path -> env list =
  fun prog (glist, env) = match glist with
  | [] -> [env]
  | goal::goals ->
    let envs = call prog env goal in
    let newpaths = map (fun e -> (goals, e)) envs
    in (flatten (map (exec_path prog) newpaths))
```

A Prolog interpreter to compute all solutions (Continued)

```
let find_path: goal -> env -> clause -> path =
  fun goal (bp, subst) clause =
    let (hd::body) = alloc_locals bp clause in
    try let subst' = assign_to_formals hd goal subst
        in (body, (bp+(numvars hd)+(numvarslist body), subst'))
    with Unif_fail -> ([], (-1, subst))

let assign_to_formals hd goal subst: subst = unify subst hd goal

let rec alloc_locals: int -> term list -> term list =
  fun bp ts = let alloc_local t = match t with
    | Var(i) -> Var(bp+i)
    | Nvar(c, ts) -> Nvar(c, alloc_locals bp ts)
  in map alloc_local ts
```

Implementing Backtracking

- Simply replace eager evaluation used in the interpreter with *lazy evaluation!*
- But OCaml does not support lazy evaluation
  - Use a language like Haskell that supports lazy evaluation
  - Employ a simple trick to achieve lazy evaluation in OCaml
    - The same trick can also be used in any language that supports lambda abstractions!
    - That includes C++, JavaScript, Python, ...
- Write a top-level print function that consumes the set of solutions one-at-a-time
  - prints the first solution
  - based on user input, either terminates or continues in the print/user-input loop.

Lazy Evaluation in OCaml

- *Lazy evaluation:* suspend actual parameter evaluation until needed
  - The expression is stored as a *closure* that encapsulates the binding of local variables
- *Lambda definitions* already require this ability
  - The body of the function is an expression that needs to be represented as a closure
- *Idea:* Use lambda definition f_e to represent e needing lazy evaluation

$$\texttt{fun}\ f_e()\ \texttt{->}\ e$$

  - *Note:* f_e takes an empty argument (technically, a zero-tuple, aka `unit` in OCaml)
  - Evaluation of e is suspended, until it is applied to a `unit` argument

Some types and functions for Lazy Evaluation in OCaml

- A type to represent lazily evaluated expressions

  ```
  type 'a thunk = Thunk of (unit -> 'a) | Val of 'a
  ```

- A function to force evaluation of `thunks`:

  ```
  let force v = match v with Thunk x -> x() | Val x -> x
  ```

- A variant of list type that is evaluated lazily

  ```
  type 'a lzlist = Nil | Cons of 'a * ('a lzlist thunk)
  ```

- To operate on such lazy lists, we need to redefine familiar list operations such as append, map, filter, flatten, etc.
  - But almost no other changes needed to the interpreter!

Example: Redefining `map` for `lzlist`

```
type 'a thunk = Thunk of (unit -> 'a) | Val of 'a

let rec lzmap (f: 'a -> 'b) (l: 'a lzlist): 'b lzlist =
  match l with
    | Nil -> Nil
    | Cons(l1, ls) ->
       Cons((f l1), Thunk(fun () -> map f (force ls)))
```

A Backtracking Prolog interpreter

```
let rec call: (prog: clause list) (env:env) (goal:goal): env lzlist =
 let paths = (map (find_path goal env) prog) in
 let viable_paths = filter (fun (_, (bp, _)) -> bp > 0) paths
 in exec_paths prog viable_paths

and exec_paths prog paths = match paths with
  | [] -> Nil
  | p::ps-> (lzappend (exec_path prog p) (Thunk(fun () -> (exec_paths prog ps))))

and exec_path: program -> path -> lzenv list =
 fun prog (glist, env) = match glist with
  | [] -> Cons(env, Val(Nil))
  | goal::goals ->
    let envs = call prog env goal in
    let newpaths = lzmap (fun e -> (goals, e)) envs
    in (lzflatten (lzmap (exec_path prog) newpaths))
```

Controlling Search

- **If-then-else:** Written as (c -> t ; e) where c, t, e are conjunction of atoms.

  Example:

```
gen(N, L) :-
        (N = 0
           -> L = []
           ;  M is N-1, gen(M, K), L = [N|R]).
```

Controlling Search (Contd.)

- **Pruning:** Proof search can be pruned using "`!`" (cut).
  - Cut throws away other choices when more than one clause is applicable.
  - *Use with care:* Prolog's proof process may be hard to understand, and cuts may make the program difficult to comprehend!

```
member(X, [X|_]).
member(X, [Y|Ys]) :-
        member(X, Ys).
```

Finds elements of a list.
Given `X` *and* `L`, `member(X, L)` determines whether `X` is in `L` or not.
Given `L` alone, `member(X, L)` binds `X` to elements of `L` (one by one, when backtracking).

```
member(X, [X|_]) :- !.
member(X, [Y|Ys]) :-
        member(X, Ys).
```

Finds whether or not an element is in a list.
Given `X` *and* `L`, `member(X, L)` determines whether `X` is in `L` or not.
Given `L` alone, `member(X, L)` binds `X` to the first element of `L`.

Change for a dollar

```
change([H,Q,D,N,P]) :-
           member(H,[0,1,2]), /*Half-dollars*/
           member(Q,[0,1,2,3,4]), /*quarters*/
           member(D,[0,1,2,3,4,5,6,7,8,9,10]), /* dimes */
           member(N,[0,1,2,3,4,5,6,7,8,9,10,
                         11,12,13,14,15,16,17,18,19,20]), /*nickels*/

S is 50*H+25*Q+10*D+5*N,
S=<100,
P is 100-S.
```

Permutation

```
takeout(X,[X|R],R).
takeout(X,[F|R],[F|S]) :- takeout(X,R,S).

perm([],[]).
perm([X|Y],Z) :-perm(Y,W), takeout(X,Z,W).
```

Tree Isomorphism

```
isomorphic(void, void).
isomorphic(tree(Node, Left1, Right1),
           tree(Node, Left2, Right2)) :-
     isomorphic(Left1, Left2),
     isomorphic(Right1, Right2).
isomorphic(tree(Node, Left1, Right1),
           tree(Node, Left2, Right2)) :-
     isomorphic(Left1, Right2),
     isomorphic(Right1, Left2).
```

Checking/Generating Subtrees

```
subtree(Tree1, Tree2) :-
     isomorphic(Tree1, Tree2).
subtree(Tree1, tree(Node, Left, Right)) :-
     subtree(Tree1, Left); subtree(Tree1, Right).
```

N-Queens

```
solve(P) :-
     perm([1,2,3,4,5,6,7,8],P),
     combine([1,2,3,4,5,6,7,8],P,S,D),
     all_diff(S), all_diff(D).

combine([X1|X],[Y1|Y],[S1|S],[D1|D]) :-
     S1 is X1+Y1, D1 is X1-Y1,
     combine(X,Y,S,D).
combine([],[],[],[]).

all_diff([X|Y]) :- \+member(X,Y), all_diff(Y).
all_diff([X]).
```

Merge Sort

```
merge_sort([], []).
merge_sort([X], [X]).
merge_sort(List, SortedList) :-
     split(List, First, Second),
     merge_sort(First, SortedFirst),
     merge_sort(Second, SortedSecond),
     merge(SortedFirst, SortedSecond, SortedList).

split([], [], []).
split([X], [X], []).
split([X1,X2|Xs], [X1|Ys], [X2|Zs]) :- split(Xs, Ys, Zs).
```

Merge Sort (Contd.)

```
merge([], X, X).
merge(X, [], X).
merge([X|Xs], [Y|Ys], [X|Zs]) :-
     X=<Y,
     merge(Xs, [Y|Ys], Zs).
merge([X|Xs], [Y|Ys], [Y|Zs]) :-
     X > Y,
     merge([X|Xs], Ys, Zs).
```